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MA 495/595
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Homework 8

Due March 7th on paper at the beginning of class. Please let me know if you have a question or find a mistake. There are some hints on the second page.

1. Let $f(x) = x$ for $0 < x < 1$. For this problem use Fourier series in terms of sines and cosines.
   (a) Let f_1 be the odd extension of f with period 2. Sketch f_1 and find the first four nonzero terms of its Fourier series.
   (b) Let f_2 be the even extension of f with period 2. Sketch f_2 and find the first four nonzero terms of its Fourier series.
   (c) Let f_3 be the extension of f with period 1. Sketch f_3 and find the first four nonzero terms of its Fourier series.
   (d) *This part is not to be handed in but is just for fun.* Notice how the speed at which the terms go to zero is related to the sizes of the jumps. You may enjoy taking various partial sums $S_N(x)$ and plotting $x - S_N(x)$ on Desmos to compare how well they do. The difference is significant even on intervals like $.4 \le x \le .6$ that stay well away from the endpoints. For instance, look at how many terms are needed for 1% error on such an interval.
2. Solve the heat equation $\partial_t u = \partial_x^2 u + \partial_y^2 u$ in the rectangle $R = (0, 2) \times (0, 3)$, subject to Dirichlet boundary conditions and initial condition $u(0, x, y) = 1$. Show that the solution can be written in the form $u(t, x, y) = v(t, x)w(t, y)$, where v and w each solve a heat equation in an interval.
   Again not to hand in but note that the same thing works if R is replaced by any domain (in any number of dimensions) which can be decomposed as a product, but not if the heat equation is replaced by the wave equation.

Hints:

The eigenfunctions you need to project onto are normalized versions of the following:

1. (a) $\sin(n\pi x)$, where n ranges over positive integers.
   (b) $\cos(n\pi x)$, where n ranges over nonnegative integers.
   (c) Both of the above sets, with π replaced by 2π.
2. $\sin(m\pi x/2)\sin(n\pi y/3)$, where m and n range over positive integers.

The series form of the solution of the heat equation is given in equation (8.3).